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Information transfer implies state collapse

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Abstract
We attempt to clarify certain puzzles concerning state collapse. In open quantum systems decoherence is a necessary consequence of the transfer of information to the outside; we prove an upper bound for the amount of coherence which can survive such a transfer. In large closed systems decoherence is not observed, but we will show that it is usually harmless to assume its occurrence. An independent physical collapse mechanism over and above Schrödinger's equation and the probability interpretation of quantum states is therefore redundant.

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1. Introduction

In its most basic formulation, quantum theory encodes the preparation of a system in a pure quantum state, a unit vector ψ in a Hilbert space $\mathcal{H}$. Observables are modelled by (say, nondegenerate) self-adjoint operators on $\mathcal{H}$. The expectation value of an observable A in a state ψ is given by $\langle \psi, A\psi \rangle$. If a is an eigenvalue of A and ψ_a a unit eigenvector, and information concerning A is somehow extracted from the system, then the probability for the value a to be observed is $|\langle \psi_a, \psi \rangle|^2$. If this observation is indeed made, then the subsequent behaviour of the system is predicted using the pure state ψ_a as a starting point. This is called *state collapse*. It follows that, if the information extraction has taken place but the information on the value of A is disregarded, then the subsequent behaviour can be described optimally using a mixture of eigenstates. This is called *decoherence*. In this paper we substantiate and quantify the following claim concerning decoherence:

> *Decoherence is only observed in open systems, where it is a necessary consequence of the transfer of information to the outside.*

So the observed occurrence of decoherence does not contradict the unitary time evolution postulated by quantum mechanics, since open systems do not evolve unitarily. Decoherence can be explained in quantum theory by embedding the quantum system into a larger, closed whole, which in itself evolves unitarily. This is well known (see, e.g., [Neu]). We add the observation that decoherence is not only a *possibility* for an open system, but a *necessary consequence* of the leakage of information out of the system. We prove an inequality relating the decoherence between two pure states to the degree in which a decision between the two is possible by a measurement outside. This is the content of theorem 3 in section 3.

Also, we have claimed that one *does not* actually observe decoherence in closed macroscopic systems. First of all, most of the systems that are ever observed are actually open, since it is extremely difficult to shield large systems from interaction. But more to the point, the difference between coherence and decoherence can only be seen by measuring some highly exotic 'stray observables' which are almost always forbiddingly hard to observe. And indeed, in those rare cases where experimenters have succeeded in measuring them, ordinary unitary evolution was found, not decoherence (see [Arn, Fri, Wal, Hor, Hac]).

We illustrate the latter point in section 4, where we show that the measurement of two classes of observables *cannot* reveal the difference between coherence and decoherence: a class of microscopic observables and a class of macroscopic observables. The 'stray observables' referred to above can therefore be neither microscopic nor macroscopic, which makes them so hard to observe.

In short, coherent superpositions of macroscopically distinguishable states are normal, everyday occurrences. We will give a detailed, quantitative account of why these coherent superpositions *cannot* be distinguished from the more classical incoherent superpositions in practice and can therefore always be regarded as such.

2. Abstract information extraction

Quantum phenomena are inherently stochastic. This means that, if quantum systems are prepared in identical ways, then nevertheless different events may be observed. In this paper, a quantum state will be taken to describe an ensemble of physical systems, e.g., a beam of particles. It is modelled by a normalized trace-class operator ρ on the Hilbert space. The expectation value of an observable A in the state ρ is then $\mathrm{tr}(\rho A)$.

An information extraction or measurement on a quantum state is to be considered as the partition of such an ensemble into subensembles, each subensemble corresponding to a measurement outcome. Let us, in the present section, not wonder *how* the splitting of ensembles can be described by quantum theory, but let us see what such an information extraction, if it can be done, will entail for the subsequent behaviour of the subensembles. Note that this process may serve as a part of the preparation for further experiments on the system, so that it must again lead to a state.

2.1. Information extraction

For simplicity let us assume that only two outcomes can occur, labelled 0 and 1, say with probabilities p_0 and p_1. The ensemble is then split in two parts, described by their respective states ρ_0 and ρ_1. The mappings $M_0 : \rho \mapsto p_0\rho_0$ and $M_1 : \rho \mapsto p_1\rho_1$ describe these subensembles. They form the components of the map

$$M : \rho \mapsto p_0\rho_0 \oplus p_1\rho_1, \tag{1}$$

which must be normalized, affine and positive. Indeed, normalization is the property that $p_0 + p_1 = 1$, and positivity is the requirement that states must be mapped to states. The affine property entails that for all states ρ and ϑ on the original system, and for all $\lambda \in [0, 1]$,

$$M(\lambda\rho + (1-\lambda)\vartheta) = \lambda M(\rho) + (1-\lambda)M(\vartheta).$$

This follows from the physical principle that a system which is prepared in the state ρ with probability λ and in the state ϑ with probability $1-\lambda$, say by tossing a coin, cannot be distinguished from a physical system in the state $\lambda\rho + (1-\lambda)\vartheta$. We emphasize that indeed this is a *physical* principle, not a matter of definitions. It states, for instance, that a bundle of particles having 50% spin up and 50% spin down cannot be distinguished from a bundle having 50% spin left and 50% spin right. This is a falsifiable statement.

2.2. State collapse

The above elementary observations are sufficient to prove that information extraction implies state collapse. If M distinguishes perfectly between the pure states ψ_0 and ψ_1, then of course $p_0 = 1$ in case $\rho = |\psi_0\rangle\langle\psi_0|$, and $p_1 = 1$ if $\rho = |\psi_1\rangle\langle\psi_1|$.

Proposition 1. *Let $\mathcal{T}(\mathcal{H})$ denote the space of trace-class operators on a Hilbert space $\mathcal{H}$, and let the map $M : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{H}) \oplus \mathcal{T}(\mathcal{H}) : \rho \mapsto M_0(\rho) \oplus M_1(\rho)$ be the linear extension of some normalized, affine and positive map on the states. Suppose that unit vectors ψ_0 and ψ_1 exist such that*

$$M(|\psi_0\rangle\langle\psi_0|) = M_0(|\psi_0\rangle\langle\psi_0|) \oplus 0 \qquad \textit{and} \qquad M(|\psi_1\rangle\langle\psi_1|) = 0 \oplus M_1(|\psi_1\rangle\langle\psi_1|). \tag{2}$$

Then we have $M(|\psi_0\rangle\langle\psi_1|) = M(|\psi_1\rangle\langle\psi_0|) = 0$.

Proof. The positivity of M yields $M(|\varepsilon\,\mathrm{e}^{\mathrm{i}\varphi}\psi_0 + \psi_1\rangle\langle\varepsilon\,\mathrm{e}^{\mathrm{i}\varphi}\psi_0 + \psi_1|) \geqslant 0$ as an operator inequality. In particular, the 0th component must be positive. As $M_0(|\psi_1\rangle\langle\psi_1|) = 0$, it follows that for all $\varepsilon, \varphi \in \mathbb{R}$, we have $\varepsilon^2 M_0(|\psi_0\rangle\langle\psi_0|) + \varepsilon(\mathrm{e}^{\mathrm{i}\varphi}M_0(|\psi_0\rangle\langle\psi_1|) + \mathrm{e}^{-\mathrm{i}\varphi}M_0(|\psi_1\rangle\langle\psi_0|)) \geqslant 0$. Taking the limit $\varepsilon \downarrow 0$ yields $(\mathrm{e}^{\mathrm{i}\varphi}M_0(|\psi_0\rangle\langle\psi_1|) + \mathrm{e}^{-\mathrm{i}\varphi}M_0(|\psi_1\rangle\langle\psi_0|)) \geqslant 0$ for all $\varphi \in \mathbb{R}$. In particular for $\varphi = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, implying $M_0(|\psi_0\rangle\langle\psi_1|) = M_0(|\psi_1\rangle\langle\psi_0|) = 0$.

Exchanging the roles of ψ_0 and ψ_1 in the argument above results in $M_1(|\psi_0\rangle\langle\psi_1|) = M_1(|\psi_1\rangle\langle\psi_0|) = 0$, proving the proposition. □

We may draw two conclusions from proposition 1. The first is that, for all $|\psi\rangle = \alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$, we have

$$(M_0 + M_1)(|\psi\rangle\langle\psi|) = (M_0 + M_1)(|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|). \tag{3}$$

In words, for the prediction of events *after* the splitting of the ensemble in two, it no longer matters whether *before* the splitting the system was in the pure state $|\alpha_0\psi_0 + \alpha_1\psi_1\rangle\langle\alpha_0\psi_0 + \alpha_1\psi_1|$ or in the mixed state $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$. This phenomenon, which is a direct consequence of structure (1) of the measurement process, we will call *decoherence*.

The second conclusion from proposition 1 is the following. For all $|\psi\rangle = \alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$, we have

$$M(|\psi\rangle\langle\psi|) = |\alpha_0|^2 M_0(|\psi_0\rangle\langle\psi_0|) \oplus |\alpha_1|^2 M_1(|\psi_1\rangle\langle\psi_1|). \tag{4}$$

In words, if an ensemble is split in two parts, then the '0-ensemble' will further behave as if the system had been in state ψ_0 instead of ψ prior to splitting, and the '1-ensemble' as if it had been in state ψ_1 instead of ψ. This phenomenon will be called *collapse*.

The difference between decoherence and collapse is that collapse is a statement about the connection between measurement outcome and final state, whereas decoherence only concerns the final state. In order to clarify this distinction, let us consider the map $M : \mathcal{T}(\mathbb{C}^2) \to \mathcal{T}(\mathbb{C}^2) \oplus \mathcal{T}(\mathbb{C}^2)$ defined by $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} a-c & 0 \\ 0 & b \end{pmatrix} \oplus \begin{pmatrix} c & 0 \\ 0 & d-b \end{pmatrix}$. With $\psi_0 = (1, 0)$

and $\psi_1 = (0, 1)$, it satisfies $M(|\psi_0\rangle\langle\psi_0|) = |\psi_0\rangle\langle\psi_0| \oplus 0$ and $M(|\psi_1\rangle\langle\psi_1|) = 0 \oplus |\psi_1\rangle\langle\psi_1|$. Moreover, it satisfies the decoherence relation (3). However, it does not satisfy the collapse relation (4), since $M\big(\frac{1}{2}|\psi_0+\psi_1\rangle\langle\psi_0+\psi_1|\big) = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| \oplus |\psi_0\rangle\langle\psi_0|)$, which is the wrong way round. Of course this example is unphysical: M is not positive. All it does is show that the statement that collapse occurs is strictly stronger than the statement that decoherence occurs.

Throughout this paper, we will maintain a sharp distinction between the collapse $M : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{H}) \oplus \mathcal{T}(\mathcal{H})$ and the decoherence $(M_0 + M_1) : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{H})$. The former represents the splitting of an ensemble in two parts by means of measurement, whereas the latter represents the splitting and subsequent recombination of this ensemble.

3. Open systems

A decoherence-mapping $(M_0 + M_1) : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{H})$ maps the pure state $|\alpha_0\psi_0 + \alpha_1\psi_1\rangle\langle\alpha_0\psi_0 + \alpha_1\psi_1|$ and the mixed state $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$ to the same final state. Since unitary maps preserve purity, there cannot exist a unitary map $U : \mathcal{H} \to \mathcal{H}$ such that for all $\rho \in \mathcal{T}(\mathcal{H})$:

$$(M_0 + M_1)(\rho) = U\rho U^*.$$

However, according to Schrödinger's equation the development of a closed quantum system is given by a unitary operator. We conclude that the decoherence (3) is impossible in a closed system. On the other hand decoherence is a well known and experimentally confirmed phenomenon.

We will therefore consider open systems, i.e. quantum systems which do not obey the Schrödinger equation, but are part of a larger system which does. It has often been pointed out (e.g. [Neu, Zur]) that decoherence can well occur in this situation, provided that states are only evaluated on the observables of the smaller system. We are more ambitious here: we shall prove that this form of 'local' decoherence is not just a *possible*, but an *unavoidable* consequence of information transfer out of the open system.

3.1. Unitary information transfer and decoherence

We assume that the open system has Hilbert space $\mathcal{H}$, and that its algebra of observables is given by $B(\mathcal{H})$, the bounded operators on $\mathcal{H}$. We may then assume that the larger system has Hilbert space $\mathcal{K} \otimes \mathcal{H}$, since the only way to represent $\mathcal{B}(\mathcal{H})$ on a Hilbert space is in the form $A \mapsto \mathbf{1} \otimes A$ [Tak]. We may think[1] of $\mathcal{B}(\mathcal{K})$ as the observable algebra of some ancillary system in contact with our open quantum system. In this context, $\mathcal{H}$ will be referred to as the 'open system', $\mathcal{K}$ as the 'ancilla' and $\mathcal{K} \otimes \mathcal{H}$ as the 'closed system'.

We couple the system to the ancilla during a finite time interval $[0, t]$. Let $\tau \in \mathcal{T}(\mathcal{K})$ denote the state of the ancilla at time 0, and $\rho \in \mathcal{T}(\mathcal{H})$ that of the small system. The effect of the interaction is described by a unitary operator $U : \mathcal{K} \otimes \mathcal{H} \to \mathcal{K} \otimes \mathcal{H}$, and the state of the pair at time t is given by $U(\tau \otimes \rho)U^* \in \mathcal{T}(\mathcal{K} \otimes \mathcal{H})$. For convenience, we will define the information transfer map $T : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{K} \otimes \mathcal{H})$ by $T(\rho) := U(\tau \otimes \rho)U^*$.

3.1.1. Decoherence. In the above setup, we are interested in distinguishing whether the open system $\mathcal{H}$ was in state $|\psi_0\rangle$ or $|\psi_1\rangle$ at time 0. This can be done if there exists a 'pointer observable' $B \otimes \mathbf{1}$ in the ancilla $B(\mathcal{K})$ which takes average value b_0 in state $T(|\psi_0\rangle\langle\psi_0|)$ and

[1] Sometimes it may happen, as for instance in fermionic systems, that the observables of the ancilla do not all commute with those of the open system. Also the observable algebra on $\mathcal{K}$ may be smaller than $\mathcal{B}(\mathcal{K})$, but we will neglect these complications here.

b_1 in state $T(|\psi_1\rangle\langle\psi_1|)$. By looking only at the ancilla $\mathcal{K}$ at time t, we are then able to gain information on the state of the open system $\mathcal{H}$ at time 0. We say that information is *transferred* from $\mathcal{H}$ to $\mathcal{K}$.

Under these circumstances, we wish to prove that decoherence occurs on the open system. We prepare the ground by proving the following lemma.

Lemma 2. *Let ϑ_0, ϑ_1 be unit vectors in a Hilbert space $\mathcal{L}$, and let A and B be bounded self-adjoint operators on $\mathcal{L}$ satisfying $\|[A, B]\| \leqslant \delta\|A\|\cdot\|B\|$. For $j = 0$ or 1, let $b_j := \langle\vartheta_j, B\vartheta_j\rangle$ denote the expectation and $\sigma_j^2 := \langle\vartheta_j, B^2\vartheta_j\rangle - \langle\vartheta_j, B\vartheta_j\rangle^2$ the variance of B in the state ϑ_j. Then, if $b_0 \neq b_1$,*

$$|\langle\vartheta_0, A\vartheta_1\rangle| \leqslant \frac{\delta\|B\| + \sigma_0 + \sigma_1}{|b_0 - b_1|}\|A\|.$$

Proof. Since $\|(B - b_j)\vartheta_j\|^2 = \langle\vartheta_j, (B - b_j)^2\vartheta_j\rangle = \sigma_j^2$, we have, by the Cauchy–Schwarz inequality,

$$\begin{aligned} |(b_0 - b_1)\langle\vartheta_0, A\vartheta_1\rangle| &= |\langle\vartheta_0, (A(B - b_1) - (B - b_0)A + [B, A])\vartheta_1\rangle| \\ &\leqslant \|A\|(\sigma_1 + \sigma_0) + \delta\|A\|\cdot\|B\|. \end{aligned}$$

□

Note that, for $\delta = \sigma_0 = \sigma_1 = 0$, lemma 2 merely states that commuting operators respect each other's eigenspaces. We proceed to prove that information transfer causes decoherence on the open system (see [Jan]).

Theorem 3. *Let ψ_0 and ψ_1 be mutually orthogonal unit vectors in a Hilbert space $\mathcal{H}$, and let $\tau \in \mathcal{T}(\mathcal{K})$ be a state on a Hilbert space $\mathcal{K}$. Let $U : \mathcal{K}\otimes\mathcal{H} \to \mathcal{K}\otimes\mathcal{H}$ be unitary and define $T : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{K}\otimes\mathcal{H})$ by $T(\rho) = U(\tau\otimes\rho)U^*$. Let B be a bounded self-adjoint operator on $\mathcal{K}\otimes\mathcal{H}$, and denote by b_j and σ_j^2 its expected value and variance in the state $T(|\psi_j\rangle\langle\psi_j|)$ for $j = 0, 1$. Suppose that $b_0 \neq b_1$. Then for all $\psi = \alpha_0\psi_0 + \alpha_1\psi_1$ with $|\alpha_0|^2 + |\alpha_1|^2 = 1$ and for all bounded self-adjoint operators A on $\mathcal{K}\otimes\mathcal{H}$ such that $\|[A, B]\| \leqslant \delta\|A\|\cdot\|B\|$, we have*

$$|\mathrm{tr}(T(|\psi\rangle\langle\psi|)A) - \mathrm{tr}(T(|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|)A)| \leqslant \frac{\delta\|B\| + \sigma_0 + \sigma_1}{|b_0 - b_1|}\|A\|. \tag{5}$$

Proof. First, we prove (5) in the special case that $\tau = |\varphi\rangle\langle\varphi|$ for some vector $\varphi \in \mathcal{K}$. We introduce the notation $\vartheta_j := U(\varphi\otimes\psi_j)$. Recall that the expectation of B is given by $b_j = \mathrm{tr}(T(|\psi_j\rangle\langle\psi_j|)B)$ and its variance by $\sigma_j^2 = \mathrm{tr}(T(|\psi_j\rangle\langle\psi_j|)B^2) - \mathrm{tr}^2(T(|\psi_j\rangle\langle\psi_j|)B)$. In terms of ϑ_j, this reduces to $b_j = \langle\vartheta_j, B\vartheta_j\rangle$ and $\sigma_j^2 = \langle\vartheta_j, B^2\vartheta_j\rangle - \langle\vartheta_j, B\vartheta_j\rangle^2$. Similarly, the lhs of (5) equals $|\overline{\alpha_0}\alpha_1\langle\vartheta_0, A\vartheta_1\rangle + \alpha_0\overline{\alpha_1}\langle\vartheta_1, A\vartheta_0\rangle|$, a quantity bounded by $|\langle\vartheta_0, A\vartheta_1\rangle|$ since $2|\alpha_0|\cdot|\alpha_1| \leqslant 1$. Formula (5) is then a direct application of lemma 2.

To reduce the general case to the case above, we note that a non-pure state τ can always be represented as a vector state. Explicitly, suppose that τ decomposes as $\tau = \sum_{i\in\mathbb{N}}|\beta_i|^2|\varphi_i\rangle\langle\varphi_i|$. Then define the Hilbert space $\tilde{\mathcal{K}} := \bigoplus_{i\in\mathbb{N}}\mathcal{K}_i$, where each $\mathcal{K}_i$ is a copy of $\mathcal{K}$. Now since $\left(\bigoplus_{i\in\mathbb{N}}\mathcal{K}_i\right)\otimes\mathcal{H} \cong \bigoplus_{i\in\mathbb{N}}(\mathcal{K}_i\otimes\mathcal{H})$, we may define, for each $X \in \mathcal{B}(\mathcal{K}\otimes\mathcal{H})$, the operator $\tilde{X} \in \mathcal{B}(\tilde{\mathcal{K}}\otimes\mathcal{H})$ by diagonal action on the components of the sum, i.e. $\tilde{X}\left(\bigoplus_{i\in\mathbb{N}}(k_i\otimes h_i)\right) := \bigoplus_{i\in\mathbb{N}}X(k_i\otimes h_i)$. If we now define the vector $\tilde{\varphi} \in \tilde{\mathcal{K}}$ by $\tilde{\varphi} = \bigoplus_i \beta_i\varphi_i$, then we have for all $X \in \mathcal{K}\otimes\mathcal{H}$ and $\chi \in \mathcal{H}$:

$$\begin{aligned} \mathrm{tr}(\tilde{U}(|\tilde{\varphi}\rangle\langle\tilde{\varphi}|\otimes|\chi\rangle\langle\chi|)\tilde{U}^*\tilde{X}) &= \left\langle\bigoplus_{i\in\mathbb{N}}(\beta_i\varphi_i\otimes\chi), \tilde{U}^*\tilde{X}\tilde{U}\bigoplus_{j\in\mathbb{N}}(\beta_j\varphi_j\otimes\chi)\right\rangle_{\tilde{\mathcal{K}}\otimes\mathcal{H}} \\ &= \left\langle\bigoplus_{i\in\mathbb{N}}(\beta_i\varphi_i\otimes\chi), \bigoplus_{j\in\mathbb{N}}U^*XU(\beta_j\varphi_j\otimes\chi)\right\rangle_{\tilde{\mathcal{K}}\otimes\mathcal{H}} \end{aligned}$$

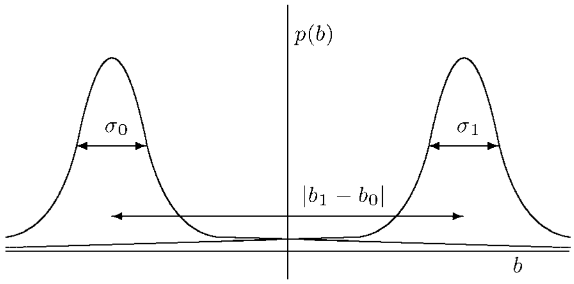

Figure 1. Probability densities p of B according to input $|\psi_0\rangle$ and $|\psi_1\rangle$.

$$
\begin{aligned}
&= \sum_{i\in\mathbb{N}} |\beta_i|^2 \langle(\varphi_i \otimes \chi), U^* X U(\varphi_i \otimes \chi)\rangle_{\mathcal{K}\otimes\mathcal{H}} \\
&= \sum_{i\in\mathbb{N}} |\beta_i|^2 \operatorname{tr}(U(|\varphi_i\rangle\langle\varphi_i| \otimes |\chi\rangle\langle\chi|)U^* X) \\
&= \operatorname{tr}(U(\tau \otimes |\chi\rangle\langle\chi|)U^* X).
\end{aligned}
$$

The second step is due to the diagonal action of the operators on $\tilde{\mathcal{K}} \otimes \mathcal{H}$. The problem is now reduced to the vector case by applying the above to $\chi = \psi$, $\chi = \psi_0$ or $\chi = \psi_1$ and on the other hand $X = A$, $X = B$ or $X = B^2$. □

The backbone of theorem 3 is formed by the special case $\sigma_0 = \sigma_1 = 0$, $[A, B] = 0$ and $\tau = |\varphi\rangle\langle\varphi|$, which allows for a short and transparent proof.

In order to arrive at a physical interpretation of theorem 3, we focus on the case $B = \tilde{B} \otimes \mathbf{1}$, when information is transferred from $\mathcal{H}$ to $\mathcal{K}$. Indeed, examining $\mathcal{K}$ at time t yields information about $\mathcal{H}$ at time 0.

3.1.2. Quality of information transfer. A small ratio $\frac{\sigma_0+\sigma_1}{|b_0-b_1|}$ indicates a good quality of information transfer. The ratio equals 0 in the perfect case, when $\sigma_0 = \sigma_1 = 0$. Thus $\tilde{B} \otimes \mathbf{1}$ takes a definite value of either b_0 or b_1, depending on whether the initial state of $\mathcal{H}$ was $|\psi_0\rangle$ or $|\psi_1\rangle$. In this case, one can infer the initial state of $\mathcal{H}$ with certainty by inspecting only the ancilla $\mathcal{K}$. More generally, it is still possible to reliably determine from the ancilla $\mathcal{K}$ whether the open system $\mathcal{H}$ was initially in state $|\psi_0\rangle$ or $|\psi_1\rangle$ as long as the standard deviations are small compared to the difference in mean, $\sigma_0, \sigma_1 \ll |b_0 - b_1|$ (figure 1).

As the ratio increases, restriction (5) gets less severe, reaching triviality at $\sigma_0 + \sigma_1 = 2|b_0 - b_1|$.

3.1.3. Decoherence on the commutant of the pointer. Assume perfect information transfer, i.e. $\sigma_0 = \sigma_1 = 0$. If $[A, B] = 0$, then theorem 3 says that coherent and mixed initial states yield identical distributions of A at time t. In order to distinguish, at time t, whether or not $\mathcal{H}$ was in a pure state at time 0, we will have to use observables A which do not commute with B. But then A and B cannot be observed simultaneously. Summarizing,

> *At time t, it is possible to distinguish whether $\mathcal{H}$ was in state ψ_0 or ψ_1 at time 0. It is also possible to distinguish whether $\mathcal{H}$ was in state ψ or $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$ at time 0. But it is not possible to do both.*

We emphasize that this holds even when one has all observables of the entire closed system $\mathcal{K} \otimes \mathcal{H}$ at one's disposal.

3.1.4. Decoherence on the open system. We consider the final state of the open system $\mathcal{H}$, obtained from the final state of the closed system $\mathcal{K} \otimes \mathcal{H}$ by tracing out the degrees of freedom of the ancilla $\mathcal{K}$: an initial state $\rho \in S(\mathcal{H})$ yields the final state $\mathrm{tr}_{\mathcal{K}}(T(\rho)) \in S(\mathcal{H})$.

Suppose that information is transferred to a pointer $B = \tilde{B} \otimes \mathbf{1}$ in the ancilla $\mathcal{K}$ with perfect quality, $\sigma_0 = \sigma_1 = 0$. Since $[\mathbf{1} \otimes \tilde{A}, \tilde{B} \otimes \mathbf{1}] = 0$, we see from theorem 3 that we have $\mathrm{tr}(T(|\psi\rangle\langle\psi|)(\mathbf{1} \otimes \tilde{A})) = \mathrm{tr}(T(|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|)(\mathbf{1} \otimes \tilde{A}))$ for *all* $\tilde{A} \in \mathcal{B}(\mathcal{H})$, or equivalently

$$\mathrm{tr}_{\mathcal{K}}(T(|\psi\rangle\langle\psi|)) = \mathrm{tr}_{\mathcal{K}}(T(|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|)). \tag{6}$$

In words,

> *Suppose that at time t, by making a hypothetical measurement of $\tilde{B}$ on the ancilla, it would be possible to distinguish perfectly whether the open system had been in state ψ_0 or ψ_1 at time 0. Then, by looking only at the observables of the open system, it is not possible to distinguish whether $\mathcal{H}$ had been in the pure state $\psi = \alpha_0\psi_0 + \alpha_1\psi_1$ or in the collapsed state $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$ at time* 0.

This statement holds true, regardless whether $\tilde{B}$ is actually measured or not. (So we do not assume here that such a measurement is physically possible.) We have shown that the map $M_0 + M_1 = \mathrm{tr}_{\mathcal{K}} \circ T$, with $T : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{K} \otimes \mathcal{H})$ the information transfer operation defined by $T(\rho) := U(\tau \otimes \rho)U^*$, constitutes a physical realization of the abstract decoherence mapping $(M_0 + M_1)$ of section 2. Decoherence is an unavoidable consequence of information transfer out of an open system.

3.1.5. Example. The simplest possible example of unitary information transfer is the following. Let $\mathcal{K} \sim \mathcal{H} \sim \mathbb{C}^2$ be the Hilbert space of a qubit; let $\psi_0 = (1, 0)$ and $\psi_1 = (0, 1)$ be the 'computational basis', and let $U : \mathbb{C}^2 \otimes \mathbb{C}^2 \to \mathbb{C}^2 \otimes \mathbb{C}^2$ be the 'controlled not-gate'. Explicitly, U is defined by $U|\psi_1 \otimes \psi_1\rangle = |\psi_0 \otimes \psi_1\rangle$, $U|\psi_0 \otimes \psi_1\rangle = |\psi_1 \otimes \psi_1\rangle$, $U|\psi_1 \otimes \psi_0\rangle = |\psi_1 \otimes \psi_0\rangle$ and $U|\psi_0 \otimes \psi_0\rangle = |\psi_0 \otimes \psi_0\rangle$. That is, it flips the first qubit whenever the second qubit is set to 1. Let τ be the 0 state of the first qubit.

Since the initial state of the second qubit can be read off from the first, this situation satisfies the hypotheses of theorem 3 with $B = \sigma_z \otimes \mathbf{1}$ and $\sigma_0 = \sigma_1 = 0$. We verify equation (6). For any state $|\psi\rangle = \alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$:

$$U|\psi_0 \otimes \psi\rangle = \alpha_0|\psi_0 \otimes \psi_0\rangle + \alpha_1|\psi_1 \otimes \psi_1\rangle := |\vartheta\rangle;$$
$$\mathrm{tr}_{\mathcal{K}}(|\vartheta\rangle\langle\vartheta|) = |\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|.$$

Thus we have $\mathrm{tr}_{\mathcal{K}}(T(|\psi\rangle\langle\psi|)) = |\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$. This agrees with equation (6), since one can easily check that $\mathrm{tr}_{\mathcal{K}}(T(|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|))$ equals $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$ as well.

3.2. Unitary information transfer and state collapse

In the context of perfect information transfer to an ancillary system, the initial states $|\psi\rangle\langle\psi|$ and $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$ lead to the same final state. This phenomenon is called decoherence.

State collapse is a stronger statement: if outcome '0' is observed, then the system will further behave as if its initial state had been ψ_0 instead of ψ. Similarly, if outcome '1' is observed, then the system will behave as if its initial state had been ψ_1.

We will therefore consider the following question: suppose that we transfer information to an ancilla $\mathcal{K}$, and then separate $\mathcal{K}$ from $\mathcal{H}$, dividing $\mathcal{H}$ into subensembles according to outcome. What states do we use to describe these subensembles?

3.2.1. Joint probability distributions. A special case of an observable is an *event* p, which in quantum mechanics is represented by an (orthogonal) projection P. The relative frequency of occurrence of p is given by $\mathbb{P}(p = 1) = \mathrm{tr}(\rho P)$.

The projection $\mathbf{1} - P$ is interpreted as 'not p'. Furthermore, if a projection Q corresponding to an observable q commutes with P, then PQ is again a projection. According to quantum mechanics, p and q can then be observed simultaneously, and the projection PQ is interpreted as the event 'p and q are both observed'.

A state ρ therefore induces a joint probability distribution on p and q:

$$\mathrm{tr}(\rho PQ) = \mathbb{P}(p = 1, q = 1), \qquad \mathbb{P}(p = 0, q = 1) = \mathrm{tr}(\rho(\mathbf{1} - P)Q)$$
$$\mathrm{tr}(\rho P(\mathbf{1} - Q)) = \mathbb{P}(p = 1, q = 0), \qquad \mathbb{P}(p = 0, q = 0) = \mathrm{tr}(\rho(\mathbf{1} - P)(\mathbf{1} - Q)).$$

Particularly relevant is the case in which ρ is a state on a combined space $\mathcal{K} \otimes \mathcal{H}$, and the projections are of the form $Q \otimes \mathbf{1}$ and $\mathbf{1} \otimes P$. (The commuting projections are properties of different systems.) We then have $\mathbb{P}(p = 1, q = 1) = \mathrm{tr}((\mathbf{1} \otimes P)(Q \otimes \mathbf{1})\rho) = \mathrm{tr}(P\, \mathrm{tr}_{\mathcal{K}}((Q \otimes \mathbf{1})\rho))$. This holds for all projections P on $\mathcal{H}$, so that the normalized version of $\mathrm{tr}_{\mathcal{K}}((Q \otimes \mathbf{1})\rho) \in \mathcal{T}(\mathcal{H})$ must be interpreted as the state of $\mathcal{H}$, given that $q = 1$. Similarly, the normalized version of $\mathrm{tr}_{\mathcal{K}}((\mathbf{1} - Q) \otimes \mathbf{1}\rho) \in \mathcal{T}(\mathcal{H})$ is the state of $\mathcal{H}$, given that $q = 0$ is observed (see, e.g., [Mac]).

3.2.2. Collapse. Let $T : \rho \mapsto U(\tau \otimes \rho)U^*$ from $\mathcal{T}(\mathcal{H})$ to $\mathcal{T}(\mathcal{K} \otimes \mathcal{H})$ be an information transfer from $\mathcal{H}$ to a pointer-projection $Q \in B(\mathcal{K})$. That is, $\mathrm{tr}((Q \otimes \mathbf{1})T(|\psi_0\rangle\langle\psi_0|)) = 0$ and $\mathrm{tr}((Q \otimes \mathbf{1})T(|\psi_1\rangle\langle\psi_1|)) = 1$, so that at time t, one can see from $\mathcal{K}$ whether $\mathcal{H}$ was in state ψ_0 or ψ_1 at time 0.

Since $Q \otimes \mathbf{1}$ commutes with all of $\mathbf{1} \otimes B(\mathcal{H})$, it is possible to separate $\mathcal{H}$ from $\mathcal{K}$ and divide $\mathcal{H}$ into subensembles according to the outcome of Q. This is done as follows: with any measurement on $\mathcal{H}$, a simultaneous measurement of Q on $\mathcal{K}$ is performed to determine in which ensemble this particular system should fall. It follows from the above that the 1-ensemble should be described by the normalized version of $M_1(\rho) := \mathrm{tr}_{\mathcal{K}}((Q \otimes \mathbf{1})T(\rho))$ and the 0-ensemble by the normalized version of $M_0(\rho) := \mathrm{tr}_{\mathcal{K}}((\mathbf{1} - Q \otimes \mathbf{1})T(\rho))$. Since Q commutes with $B(\mathcal{H})$, this is just conditioning on a classical probability space at time t. We have arrived at an interpretation of the map $M(\rho) := M_0(\rho) \oplus M_1(\rho)$ of section 2.

We will now prove that M takes the form $M(|\psi\rangle\langle\psi|) = |\alpha_0|^2\, \mathrm{tr}_{\mathcal{K}} T(|\psi_0\rangle\langle\psi_0|) \oplus |\alpha_1|^2\, \mathrm{tr}_{\mathcal{K}} T(|\psi_1\rangle\langle\psi_1|)$.

One could alternatively, (and more traditionally), arrive at the 'collapse of the wavefunction' $M(|\psi\rangle\langle\psi|) = |\alpha_0|^2\, \mathrm{tr}_{\mathcal{K}} T(|\psi_0\rangle\langle\psi_0|) \oplus |\alpha_1|^2\, \mathrm{tr}_{\mathcal{K}} T(|\psi_1\rangle\langle\psi_1|)$ by assuming that, at time 0, the quantum system makes either the jump $|\psi\rangle\langle\psi| \mapsto |\psi_0\rangle\langle\psi_0|$ or the jump $|\psi\rangle\langle\psi| \mapsto |\psi_1\rangle\langle\psi_1|$. Since we arrive at the same conclusion, namely the above 'collapse of the wavefunction', using only open systems, unitary transformations and the probabilistic interpretation of quantum mechanics, such a retrospective assumption of 'jumps' at time 0 is made redundant.

Proposition 4. *Let $T : \rho \mapsto U(\tau \otimes \rho)U^*$ from $\mathcal{T}(\mathcal{H})$ to $\mathcal{T}(\mathcal{K} \otimes \mathcal{H})$ satisfy $\mathrm{tr}((Q \otimes \mathbf{1})T(|\psi_0\rangle\langle\psi_0|)) = 0$ and $\mathrm{tr}((Q \otimes \mathbf{1})T(|\psi_1\rangle\langle\psi_1|)) = 1$ for some 'pointer-projection' Q on $\mathcal{K}$. Define a map $M : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{H}) \oplus \mathcal{T}(\mathcal{H})$ by $M(\rho) := \mathrm{tr}_{\mathcal{K}}((\mathbf{1} - Q \otimes \mathbf{1})T(\rho))$*

$\oplus \operatorname{tr}_{\mathcal{K}}((Q \otimes \mathbf{1})T(\rho))$. *Then for* $\psi = \alpha_0\psi_0 + \alpha_1\psi_1$, *we have* $M(|\psi\rangle\langle\psi|) = |\alpha_0|^2 \operatorname{tr}_{\mathcal{K}} T(|\psi_0\rangle\langle\psi_0|) \oplus |\alpha_1|^2 \operatorname{tr}_{\mathcal{K}} T(|\psi_1\rangle\langle\psi_1|)$.

This can be seen almost directly from proposition 1.

Proof. Since $M_1(|\psi_0\rangle\langle\psi_0|) \geqslant 0$ is a positive operator, we may conclude from $\operatorname{tr}(M_1(|\psi_0\rangle\langle\psi_0|)) = 0$ that $M_1(|\psi_0\rangle\langle\psi_0|) = 0$. Similarly $M_0(|\psi_1\rangle\langle\psi_1|) = 0$. Utilizing proposition 1, we find that $M(|\psi\rangle\langle\psi|) = |\alpha_0|^2 M_0(|\psi_0\rangle\langle\psi_0|) \oplus |\alpha_1|^2 M_1(|\psi_1\rangle\langle\psi_1|)$. The proof is completed by noting from $\operatorname{tr}_{\mathcal{K}}((\mathbf{1} - Q \otimes \mathbf{1})T(|\psi_1\rangle\langle\psi_1|)) = 0$ that $\operatorname{tr}_{\mathcal{K}}(T(|\psi_1\rangle\langle\psi_1|)) = \operatorname{tr}_{\mathcal{K}}((Q \otimes \mathbf{1})T(|\psi_1\rangle\langle\psi_1|)) = M_1(|\psi_1\rangle\langle\psi_1|)$ and similarly that $\operatorname{tr}_{\mathcal{K}}(T(|\psi_0\rangle\langle\psi_0|)) = M_0(|\psi_0\rangle\langle\psi_0|)$. □

We summarize as follows:

> *Consider an ensemble of systems of type $\mathcal{H}$ in state ψ. Suppose that information is transferred to a pointer-projection Q on an ancillary system $\mathcal{K}$. Subsequently, the ensemble is divided into two subensembles according to outcome. Then all observations on $\mathcal{H}$ made afterwards, conditioned on the observation that the measurement outcome was 0, will be as if the system had originally been in the collapsed state ψ_0 instead of ψ. No independent physical collapse mechanism is needed to arrive at this conclusion.*

3.2.3. Example. In the simple model of information transfer introduced in section 3.1, we will now demonstrate why repeated spin measurements yield identical outcomes.

The probed system is once again a single spin $\mathcal{H} = \mathbb{C}^2$, whereas the ancillary system now consists of two spins, $\mathcal{K} = \mathbb{C}^2 \otimes \mathbb{C}^2$ in initial state $|\psi_0 \otimes \psi_0\rangle$. Repeated information transfer, first to pointer $\sigma_{z,1} := \sigma_z \otimes \mathbf{1}$ and then to $\sigma_{z,2} := \mathbf{1} \otimes \sigma_z$, is then represented by the unitary $U := U_2U_1$ on $\mathcal{K} \otimes \mathcal{H}$. In this expression, U_1 is the controlled not-gate flipping the first qubit of $\mathcal{K}$ if $\mathcal{H}$ is set to 1, and U_2 flips the second qubit of $\mathcal{K}$ if $\mathcal{H}$ is set to 1.

Since $U|\psi_0 \otimes \psi_0 \otimes (\alpha_0\psi_0 + \alpha_1\psi_1)\rangle = |\alpha_0\psi_0 \otimes \psi_0 \otimes \psi_0\rangle + |\alpha_1\psi_1 \otimes \psi_1 \otimes \psi_1\rangle$, we can explicitly calculate the joint probability distribution on the two pointers $\sigma_{z,1}$ and $\sigma_{z,2}$ in the final state:

$$\begin{aligned} &\mathbb{P}(s_{z,1} = 1, s_{z,2} = 1) = |\alpha_1|^2, && 0 = \mathbb{P}(s_{z,1} = 1, s_{z,2} = -1) \\ &\mathbb{P}(s_{z,1} = -1, s_{z,2} = 1) = 0, && |\alpha_0|^2 = \mathbb{P}(s_{z,1} = -1, s_{z,2} = -1). \end{aligned}$$

In particular, we see that if the first outcome is 1 (which happens with probability $|\alpha_1|^2$), then so is the second. Proposition 4 shows that this is the general situation, independent of the (rather simplistic) details of this particular model.

3.3. Information leakage to the environment

On closed systems decoherence does not occur, because unitary time evolution preserves the purity of states. However, macroscopic systems are almost never closed.

Imagine, for example, that $\mathcal{H} = \mathbb{C}^2$ represents a two-level atom and $\mathcal{K}$ some large measuring device. Information about the energy $\mathbf{1} \otimes \sigma_z$ of the atom is transferred to the apparatus, where it is stored as the position $\tilde{B} \otimes \mathbf{1}$ of a pointer. Then as soon as information on the pointer-position $\tilde{B} \otimes \mathbf{1}$ leaves the system, decoherence on the combined atom–apparatus system takes place. For example, a ray of light may reflect on the pointer, revealing its position to the outside world (see [JZ, Kok]). It is of course immaterial whether or not someone is actually *looking* at the photons. If even the smallest speck of light were to fall on the pointer, the information about the pointer position would already be encoded in the light, causing full decoherence on the atom–apparatus system.

Similarly, emission of thermal radiation and collisions with gas-particles transfer information to the environment, causing decoherence [Hac, Hor]. Coupling to a bath of oscillators, as modelled in [Zur], also fits this profile.

The quality of this information transfer will not be perfect. If a macroscopic system is interacting normally with the outside world, (the occasional photon scatters on it, for instance), then a number of macroscopic observables X will leak information continually, with a macroscopic uncertainty σ. This enables us to apply theorem 3. It says that all coherences between eigenstates ψ_{x_1} and ψ_{x_2} of macroscopic observables X are continually vanishing on the macroscopic system $\mathcal{L}$, provided that their eigenvalues x_1 and x_2 satisfy $|x_1 - x_2| \gg 2\sigma$. (The pointer, e.g. a beam of light, is outside the system, so that $\delta = 0$.)

Take for example a collection of N spins, $\mathcal{L} = \bigotimes_{i=1}^{N} \mathbb{C}^2$. Suppose that for $\alpha = x, y, z$, the average spin observables $S_\alpha = \frac{1}{N}\sum_{i=1}^{N} \sigma_\alpha^i$ are continually being measured with an accuracy[2] $N^{-\frac{1}{2}} \ll \sigma \ll 1$. Then between macroscopically different eigenstates of S_α, i.e. states for which the eigenvalues satisfy $|s_\alpha - s'_\alpha| \gg \sigma$, coherences are constantly disappearing. However, the information leakage need not have any effect on states which only differ on a microscopic scale. Take for instance $\rho \otimes |+\rangle\langle+|$ and $\rho \otimes |-\rangle\langle-|$, with ρ an arbitrary state on $N-1$ spins. Indeed, $|s_\alpha - s'_\alpha| \leqslant 2/N \ll \sigma$, so theorem 3 is vacuous in this case: no decoherence occurs.

We see how the variance σ^2 produces a smooth boundary between the macroscopic and the microscopic world: macroscopically distinguishable states (involving S_α-differences $\gg \sigma$) continually suffer from loss of coherence, while states that only differ microscopically (involving S_α-differences $\ll \sigma$) are unaffected.

In case of a system monitored by a macroscopic measurement apparatus, we are interested in coherence between eigenstates of the macroscopic pointer. By definition, these eigenstates are macroscopically distinguishable. We may then give the following answer to the question why it is so hard, in practice, to witness coherence:

> *If information leaks from the pointer into the outside world, decoherence takes place on the combination of system and measurement apparatus. In practice, macroscopic pointers constantly leak information.*

4. Closed systems

Closed systems evolve according to unitary time evolution, so that the coherence which is present initially will still be there at later times. Yet on macroscopic systems, coherent superpositions are almost never observed. Why is this the case?

4.1. Macroscopic systems

Because of the direct link that it provides between the scale of a system on the one hand and on the other hand the difficulties in witnessing coherence, we feel that the following line of reasoning, essentially due to Hepp [Hep], is the most important mechanism hiding coherence.

Let us first define what we mean by macroscopic and microscopic observables. We consider a system consisting of N distinct subsystems, i.e. $\mathcal{K} = \bigotimes_{i=1}^{N} \mathcal{K}_i$. If one thinks of $\mathcal{K}_i$

[2] Since $[S_x, S_y] \neq 0$, they cannot be simultaneously measured with complete accuracy, see, e.g., [Wer]. However, this problem disappears if the accuracy satisfies $\sigma^2 \geqslant \frac{1}{2}\|[S_x, S_y]\| = \frac{1}{N}$, see [Jan]. For large N, (typically $N \sim 6 \times 10^{23}$), this allows for extremely accurate measurement.

as the atoms out of which a macroscopic system $\mathcal{K}$ is constructed, N may well be in the order of 10^{23}.

We will define the *microscopic* observables to be those that refer only to one particular subsystem $\mathcal{K}_i$.

Definition. *An observable $X \in B(\mathcal{K})$ is called microscopic if it is of the form $X = \mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes X_i \otimes \mathbf{1} \otimes \cdots \otimes \mathbf{1}$ for some $i \in \{1, 2, \ldots, N\}$ and some $X_i \in B(\mathcal{K}_i)$.*

In this situation we will identify $X_i \in B(\mathcal{K}_i)$ with $X \in B(\mathcal{K})$. We take macroscopic observables to be averages of microscopic observables 'of the same size':

Definition. *An observable $Y \in B(\mathcal{K})$ is called macroscopic if it is of the form $Y = \frac{1}{N}\sum_{i=1}^{N} Y_i$, with $Y_i \in B(\mathcal{K}_i)$ such that $\|Y_i\| \leqslant \|Y\|$.*

We will only use the term 'macroscopic' in this narrow sense from here on, even though there do exist observables which are called 'macroscopic' in daily life, but do not fall under the above definition.

Now suppose that we transfer information from a system $\mathcal{H}$ to a macroscopic system $\mathcal{K} = \bigotimes_{i=1}^{N} \mathcal{K}_i$, using a macroscopic pointer $\tilde{B} \in B(\mathcal{K})$. As explained before, we then have a map $T : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{K} \otimes \mathcal{H})$ such that the pointer $\tilde{B} \otimes \mathbf{1}$ has different expectation values b_0 and b_1 in the states $T(|\psi_0\rangle\langle\psi_0|)$ and $T(|\psi_1\rangle\langle\psi_1|)$.

Since $\tilde{B}$ is macroscopic, it is unrealistic to require $T(|\psi_0\rangle\langle\psi_0|)$ and $T(|\psi_1\rangle\langle\psi_1|)$ to be eigenstates of $\tilde{B}$. Instead, we will require their standard deviations in $\tilde{B}$ to be negligible compared to their difference in mean, i.e. $\sigma_0 \ll |b_0 - b_1|$ and $\sigma_1 \ll |b_0 - b_1|$.

After this information transfer, we try to distinguish whether the system $\mathcal{H}$ had initially been in the coherent state $\alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$ or in the incoherent mixture $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$. We have already shown that this cannot be done by measuring observables in $\mathbf{1} \otimes B(\mathcal{H})$. The following adaptation of theorem 3 shows that it is also impossible to do this by measuring macroscopic or microscopic observables on the closed system $\mathcal{K} \otimes \mathcal{H}$.

Corollary 5. *Let ψ_0 and ψ_1 be orthogonal unit vectors in a Hilbert space $\mathcal{H}$ and $\tau \in \mathcal{T}(\mathcal{K})$ be a state on the Hilbert space $\mathcal{K} = \bigotimes_{i=1}^{N} \mathcal{K}_i$. Let $U : \mathcal{K} \otimes \mathcal{H} \to \mathcal{K} \otimes \mathcal{H}$ be unitary and define $T : \mathcal{T}(\mathcal{H}) \to \mathcal{T}(\mathcal{K} \otimes \mathcal{H})$ by $T(\rho) = U(\tau \otimes \rho)U^*$. Let $\tilde{B}$ be a macroscopic observable in $B(\mathcal{K})$ and define $B := \tilde{B} \otimes \mathbf{1}$. Denote by b_j and σ_j^2 its expected value and variance in the state $T(|\psi_j\rangle\langle\psi_j|)$ for $j = 0, 1$. Suppose that $b_0 \neq b_1$. Then for all $\psi = \alpha_0\psi_0 + \alpha_1\psi_1$ with $|\alpha_0|^2 + |\alpha_1|^2 = 1$ and for all microscopic and macroscopic observables $A \in B(\mathcal{K} \otimes \mathcal{H})$, we have*

$$\left|\mathrm{tr}(T(|\psi\rangle\langle\psi|)A) - \mathrm{tr}\big(T(|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + \left|\alpha_1^2\right||\psi_1\rangle\langle\psi_1|)A\big)\right| \leqslant \frac{\frac{2}{N}\|B\| + \sigma_0 + \sigma_1}{|b_0 - b_1|}\|A\|.$$

Proof. If A is microscopic, we have $\|[A, B]\| = \left\|\left[A_i, \frac{1}{N}\sum_{j=1}^{N} B_j\right]\right\| = \frac{1}{N}\|[A_i, B_i]\| \leqslant \frac{2\|A\|\|B\|}{N}$. Alternatively, if A is macroscopic, we have $\|[A, B]\| = \left\|\left[\frac{1}{N+1}\sum_{i=0}^{N} A_i, \frac{1}{N}\sum_{j=1}^{N} B_j\right]\right\| = \frac{1}{N(N+1)}\sum_{i=1}^{N}\|[A_i, B_i]\| \leqslant \frac{2\|A\|\|B\|}{N}$. Either way, we can now apply theorem 3. □

4.2. Examples

In order to illustrate the above, we discuss four examples of information transfer to a macroscopic system.

4.2.1. The finite spin-chain. We study a single spin $\mathcal{H} = \mathbb{C}^2$ in interaction with a large but finite spin chain $\mathcal{K} = \bigotimes_{i=1}^{N} \mathbb{C}^2$; the latter acting as a measurement apparatus. Once again, let $\psi_0 = (1, 0)$ and $\psi_1 = (0, 1)$ be the 'computational basis'. Initially, all spins in the spin chain are down: $\tau = |\psi_0 \otimes \cdots \otimes \psi_0\rangle\langle\psi_0 \otimes \cdots \otimes \psi_0|$. Let $U_i : \mathcal{K} \otimes \mathcal{H} \to \mathcal{K} \otimes \mathcal{H}$ be the 'controlled not-gate', which flips spin number i in the chain whenever the single qubit is set to 1. (We define $U_j = \mathbf{1}$ for $j \notin \{1, 2, \ldots, N\}$):

$$U_i = \mathbf{1} \otimes P_- + \sigma_{x,i} \otimes P_+ \qquad \text{with} \qquad P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

In discrete time $n \in \mathbb{Z}$, the unitary evolution is given by $n \mapsto U_n U_{n-1} \cdots U_2 U_1$ (see [Hep]). This represents a single spin flying over a spin chain from 1 to N, interacting with spin n at time n.

Obviously $U_N|\psi_0 \otimes \cdots \otimes \psi_0\rangle \otimes |\psi_0\rangle = |\psi_0 \otimes \cdots \otimes \psi_0\rangle \otimes |\psi_0\rangle$ and $U_N|\psi_0 \otimes \cdots \otimes \psi_0\rangle \otimes |\psi_1\rangle = |\psi_1 \otimes \cdots \otimes \psi_1\rangle \otimes |\psi_1\rangle$. We consider the average spin of the spin chain as pointer, $B = \frac{1}{N}\sum_{i=1}^{N} \sigma_{z,i}$. This makes the map $T : \rho \mapsto U_N \tau \otimes \rho U_N^*$ an information transfer to a macroscopic system. Applying corollary 5 with $b_0 = -1, b_1 = 1$ and $\sigma_0 = \sigma_1 = 0$ yields the estimate

$$\left|\mathrm{tr}(T(|\alpha_0\psi_0 + \alpha_1\psi_1\rangle\langle\alpha_0\psi_0 + \alpha_1\psi_1|)A) - \mathrm{tr}\big(T\left(|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + \left|\alpha_1^2\right||\psi_1\rangle\langle\psi_1|\right)A\big)\right| \leqslant \frac{1}{N}\|A\|$$

for all microscopic and macroscopic $A \in B(\mathcal{K} \otimes \mathcal{H})$. Indeed, in this particular model, the estimated quantity is identically zero since $\langle\psi_0 \otimes \cdots \otimes \psi_0, X_i\psi_1 \otimes \cdots \otimes \psi_1\rangle = \langle\psi_0, \psi_1\rangle^{N-1}\langle\psi_0, X_i\psi_1\rangle = 0$ for all microscopic X_i.

Of course coherence *can* be detected on the closed system $\mathcal{K} \otimes \mathcal{H}$, but only using observables that are neither macroscopic nor microscopic, such as $\sigma_x \otimes \cdots \otimes \sigma_x$.

4.2.2. Finite spin chain at nonzero temperature. A more realistic initial state for the spin chain is the nonzero-temperature state $\tau_\beta = \frac{\mathrm{e}^{-\beta H}}{\mathrm{tr}\,\mathrm{e}^{-\beta H}}$. For the spin-chain Hamiltonian we will take $H = \sum_i \sigma_{z,i} = NB$, so that τ_β becomes the tensor product of N copies of the $\mathbb{C}^2$ state

$$\hat{\tau}_\beta = \frac{1}{\mathrm{e}^{\beta} + \mathrm{e}^{-\beta}} \begin{pmatrix} \mathrm{e}^{-\beta} & 0 \\ 0 & \mathrm{e}^{\beta} \end{pmatrix}.$$

With the same time evolution as before, we have $T(|\psi_0\rangle\langle\psi_0|) = \tau_\beta \otimes |\psi_0\rangle\langle\psi_0|$ and $T(|\psi_1\rangle\langle\psi_1|) = \tau_{-\beta} \otimes |\psi_1\rangle\langle\psi_1|$. Again we choose the mean energy B as our pointer. A brief calculation shows that $\mathrm{tr}(B\tau_\beta) = \frac{\mathrm{e}^{-\beta} - \mathrm{e}^{\beta}}{\mathrm{e}^{\beta} + \mathrm{e}^{-\beta}} =: \varepsilon(\beta)$ and that $\mathrm{tr}(B^2\tau_\beta) - \mathrm{tr}(B\tau_\beta)^2 = \frac{1}{N}(1 - \varepsilon^2(\beta))$. Corollary 5 now gives us, for microscopic and macroscopic A,

$$\begin{aligned} &\left|\mathrm{tr}(T(|\alpha_0\psi_0 + \alpha_1\psi_1\rangle\langle\alpha_0\psi_0 + \alpha_1\psi_1|)A) - \mathrm{tr}\big(T\left(|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1^2||\psi_1\rangle\langle\psi_1|\right)A\big)\right| \\ &\qquad \leqslant \left(\frac{1}{\varepsilon(\beta)N} + \frac{\sqrt{1 - \varepsilon^2(\beta)}}{\varepsilon(\beta)\sqrt{N}}\right)\|A\|. \end{aligned}$$

For large N, we see that the term $\sim\frac{1}{N}$ due to the fact that $[A, B] \neq 0$ is dominated by the thermodynamical fluctuations, which of course go as $\sim\frac{1}{\sqrt{N}}$. In statistical physics, it is standard practice to neglect even the latter.

4.2.3. Energy as a pointer. Hamiltonians often fail to be macroscopic in our narrow sense of the word, since they are generically unbounded and contain interaction terms. However, this does not imply failure of our scheme to estimate coherence.

For example, consider an N-particle system with Hilbert space $\mathcal{K} = \bigotimes_{i=1}^{N} \mathcal{K}_i$ and Hamiltonian $H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + V(x_1, x_2, \ldots, x_N)$. Information is transferred from $\mathcal{H}$ to $\mathcal{K}$ with H as pointer, that is the two states $\mathrm{tr}_{\mathcal{H}}(T(|\psi_0\rangle\langle\psi_0|))$ and $\mathrm{tr}_{\mathcal{H}}(T(|\psi_1\rangle\langle\psi_1|))$ have different energies E and E'. Without loss of generality, assume that they are vectorstates: $\mathrm{tr}_{\mathcal{H}}(T(|\psi_0\rangle\langle\psi_0|)) = |\psi\rangle\langle\psi|$ and $\mathrm{tr}_{\mathcal{H}}(T(|\psi_1\rangle\langle\psi_1|)) = |\psi'\rangle\langle\psi'|$. (Density matrices can always be represented as vectors on a different Hilbert space, cf the proof of theorem 3.)

We thus have two vector states $|\psi\rangle$ and $|\psi'\rangle$ with different energies $E := \langle\psi, H\psi\rangle$ and $E' := \langle\psi', H\psi'\rangle$. We estimate the coherence between $|\psi\rangle$ and $|\psi'\rangle$ on x_n, the position of particle n

$$\begin{aligned}(E - E')\langle\psi, x_n\psi'\rangle &= \langle E\psi, x_n\psi'\rangle - \langle x_n\psi, E'\psi'\rangle \\ &= \langle H\psi - (H - E)\psi, x_n\psi'\rangle - \langle x_n\psi, H\psi' - (H - E')\psi'\rangle \\ &= \langle [H, x_n]\psi, \psi'\rangle - \langle (H - E)\psi, x_n\psi'\rangle + \langle x_n\psi, (H - E')\psi'\rangle.\end{aligned}$$

Now since $[H, x_n] = \frac{1}{2m_n}\left[p_n^2, x_n\right] = \frac{-\mathrm{i}\hbar p_n}{m_n}$, we can apply the Cauchy–Schwarz inequality in each term to obtain $|E - E'||\langle\psi, x_n\psi'\rangle| \leqslant \frac{\hbar}{m_n}\sqrt{\langle\psi, p_n^2\psi\rangle} + \sqrt{\langle\psi, x_n^2\psi\rangle}\sqrt{\langle\psi', (H - E')^2\psi'\rangle} + \sqrt{\langle\psi', x_n^2\psi'\rangle}\sqrt{\langle\psi, (H - E)^2\psi\rangle}$. If we define the characteristic speed $V_n := \sqrt{\langle\psi, \left(\frac{p_n}{m_n}\right)^2\psi\rangle}$, the characteristic positions $X_n := \sqrt{\langle\psi, x_n^2\psi\rangle}$ and $X_n' := \sqrt{\langle\psi', x_n^2\psi'\rangle}$, and the standard deviations $\sigma := \sqrt{\langle\psi, (H - E)^2\psi\rangle}$ and $\sigma' := \sqrt{\langle\psi', (H - E')^2\psi'\rangle}$, we obtain

$$|\langle\psi, x_n\psi'\rangle| \leqslant \frac{\hbar V_n + \sigma X_n' + \sigma' X_n}{|E - E'|}.$$

As such, this does not tell us very much. We will have to make some physically plausible assumptions on the state of the system in order to obtain results. First, we assume that the system is encased in an $L \times L \times L$ box so that $X_n, X_n' \leqslant L$. Also, we assume $V_n < c$. This yields $|\langle\psi, x_n\psi'\rangle| \leqslant \frac{\hbar c + L(\sigma + \sigma')}{|E - E'|}$. Secondly, we assume that scaling the system in any meaningful way will produce $|E - E'| \sim N$ and $\sigma + \sigma' \sim \sqrt{N}$, so that the coherence on x_n approaches zero as $\sim\frac{1}{\sqrt{N}}$. Note the almost thermodynamic lack of detail required for this estimate.

4.2.4. Schrödinger's cat. Let us finally analyse the rather drastic extraction of information from a radioactive particle that has become known[3] as 'Schrödinger's cat' (see [Sch]). The experiment is performed as follows. We are interested in a radioactive particle. Is it in a decayed state ψ_0 or in a non-decayed state ψ_1?

In order to determine this, we set up the following experiment. A Geiger counter is placed next to the radioactive particle. If the particle decays, then the Geiger counter clicks. A mechanism then releases a hammer, which smashes a vial of hydrocyanic acid, killing a cat. All of this happens in a closed box not higher than 1 m and completely impenetrable to information. A measurement of the atom is done as follows. First, place it inside the box. Then wait for a period of time that is long compared to the decay time of the atom. Finally, open the box and inspect whether the cat has dropped dead or is still standing upright.

The atom is described by a Hilbert space $\mathcal{H}$, the combination of Geiger counter, mechanism, hammer, vial and cat by a Hilbert space $\mathcal{K}$. Initially, the latter is prepared in a state $|\vartheta\rangle$. As a pointer, we take the centre of mass of the cat, $Z := \frac{1}{N}\sum_{i=1}^{N} z_i$. In

[3] Actually, Schrödinger's proposal was slightly different. In the original thought experiment, death of the cat was correlated with decay of the atom at time t instead of 0, which wouldn't make it an information transfer in our sense of the word.

this expression, N is the amount of atoms out of which the cat is constructed and z_i is the z component of particle number i. (It is a harmless assumption that all atoms in the cat have the same mass.) Since the box only measures 1 m in height, we may take $\|Z\| = 1$. The unitary evolution $U \in B(\mathcal{K} \otimes \mathcal{H})$ then produces $U|\psi_0 \otimes \vartheta\rangle := |\gamma_0\rangle$ and $U|\psi_1 \otimes \vartheta\rangle := |\gamma_1\rangle$, which are eigenstates[4] of Z with different eigenvalues.

Suppose that, initially, the atom is either in the decayed state ψ_0 with probability $|\alpha_0|^2$ or in the non-decayed state ψ_1 with probability $|\alpha_1|^2$. That is, the initial state is the incoherent mixture $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$. By linearity, the final state is then the incoherent state $|\alpha_0|^2|\gamma_0\rangle\langle\gamma_0| + |\alpha_1|^2|\gamma_1\rangle\langle\gamma_1|$.

On the other hand, if the atom starts out in the coherent superposition $\alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$, then the combined system ends up in the coherent state $|(\alpha_0\psi_0 + \alpha_1\psi_1) \otimes \vartheta\rangle = \alpha_0|\gamma_0\rangle + \alpha_1|\gamma_1\rangle$.

The question is now this: why do we not note the difference between these two situations if we open the box? First of all, according to theorem 3 (and the observations following it in section 3.1.3), it is impossible to detect coherence between γ_0 and γ_1 *and* ascertain the position of the cat. Upon opening the black box, we must make a choice.

Secondly, according to the discussion in section 3.3, the coherences between the macroscopically different states γ_0 and γ_1 are extremely volatile. Any speck of light falling on the cat will reveal its position with reasonable accuracy, causing the coherence to disappear according to theorem 3.

Yet even if we were able to open the box without any information on the position of the cat leaking out, even then would we be unable to detect coherence between γ_0 and γ_1. Apply corollary 5 to the transfer of information from atom to cat. We have $\sigma_0 = \sigma_1 = 0$, and with pointer Z we have $\|Z\| = 1$ (the height of the box is 1 m) and $z_1 - z_0 = 0.1$ (the difference between a cat that is standing up and one that has dropped dead is 10 cm). We then obtain for all macroscopic and microscopic A:

$$|\langle\alpha_0\gamma_0 + \alpha_1\gamma_1, A\alpha_0\gamma_0 + \alpha_1\gamma_1\rangle - (|\alpha_0|^2\langle\gamma_0, A\gamma_0\rangle + |\alpha_1|^2\langle\gamma_1, A\gamma_1\rangle)| \leqslant \frac{20}{N}\|A\|.$$

On the subset of observables we are normally able to measure the distinction between coherent and incoherent mixtures practically vanishes for $N \sim 10^{23}$. For all practical intents and purposes, it is completely harmless to assume that the final state of the cat is $|\alpha_0|^2|\gamma_0\rangle\langle\gamma_0| + |\alpha_1|^2|\gamma_1\rangle\langle\gamma_1|$ instead of $\alpha_0|\gamma_0\rangle + \alpha_1|\gamma_1\rangle$. But it would be false to state that the former has actually been observed.

5. Conclusion

In open systems, we have proven that decoherence is a necessary consequence of information transfer to the outside. More in detail, we have reached the following conclusions:

- Suppose that an open system $\mathcal{H}$ interacts with an ancillary system $\mathcal{K}$ in such a way that it is possible, in principle, to determine from $\mathcal{K}$ whether $\mathcal{H}$ had been in state ψ_0 or ψ_1 before the interaction. If $\mathcal{H}$ started out in a coherent state $\alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$, then it will behave after the information transfer as if it had started out in the incoherent mixture $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$ instead. This is called 'decoherence'.
- Suppose again that the information whether $\mathcal{H}$ was in state ψ_0 or ψ_1 is transferred to an ancillary system $\mathcal{K}$. This is done with an ensemble of $\mathcal{H}$ systems described by the state $\alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$. The ensemble is then split into subensembles, according to outcome. The '0-ensemble' then behaves as if it had been in state ψ_0 at the beginning of the

[4] As discussed before, it would be more realistic to allow for a nonzero variance $0 < \sigma_j \ll 1$ instead of requiring ϑ_j to be eigenstates of Z. We use $\sigma_j = 0$ for clarity, leaving the argument essentially unchanged.

procedure, and the '1-ensemble' as if it had started in state ψ_1. This is called 'state collapse'.

- These results were obtained entirely within the framework of traditional quantum mechanics and unitary time evolution on a larger, closed system containing $\mathcal{H}$. No physical collapse mechanism is needed. From proposition 1, we see that any information extraction causes collapse, quite independent of its particular mechanism.
- On the closed system containing the smaller, open one no decoherence occurs in principle. In practice, however, closed systems are very hard to achieve. We have argued that information transfer from a macroscopic observable A, performed with macroscopic precision σ, causes decoherence between eigenstates of A if their values satisfy $\sigma \ll |a_1 - a_0|$. Since information on macroscopic observables tends to leak out, coherence between macroscopically different states tends to vanish.

Still, even if the combined system $\mathcal{K} \otimes \mathcal{H}$ is considered perfectly closed, there are some results to be obtained. Again, we investigated the case that a system $\mathcal{H}$ interacts unitarily with a system $\mathcal{K}$ in such a way that the information whether $\mathcal{H}$ was in state ψ_0 or ψ_1 can be read off from a pointer in $\mathcal{K}$. We have reached the following conclusions concerning the closed system $\mathcal{K} \otimes \mathcal{H}$:

- Using only observables on the closed system that commute with the pointer, it is impossible to detect whether $\mathcal{H}$ had started out in state $\alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$ or $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$. Physically, this means that it is impossible to distinguish between coherent and incoherent initial states while at the same time distinguishing between ψ_0 and ψ_1.
- Suppose that the closed system $\mathcal{K} \otimes \mathcal{H}$ is macroscopic, and that one has access to its macroscopic and microscopic observables only. Then it is almost impossible to distinguish whether $\mathcal{H}$ had started out in state $\alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$ or $|\alpha_0|^2|\psi_0\rangle\langle\psi_0| + |\alpha_1|^2|\psi_1\rangle\langle\psi_1|$. We have obtained upper bounds on the coherences $\langle\psi_0, A\psi_1\rangle$, evaluated on microscopic or macroscopic A. Assuming perfect information transfer ($\sigma_0 = \sigma_1 = 0$), they approach zero as $\sim\frac{1}{N}$, where N is the size of the system.

In short: no decoherence ever occurs on perfectly closed systems, even if they are macroscopic. It is just very hard to distinguish coherent from incoherent states, creating the false *impression* of decoherence.

The link between decoherence and macroscopic systems was brought forward by Klaus Hepp in his fundamental paper [Hep], where he considered infinite closed systems, displaying decoherence in infinite time. In infinite systems, the microscopic observables form a non-commutative C*-algebra $\mathcal{A}$. Its weak closure $\mathcal{A}''$ is considered as the (von Neumann-)algebra of all observables. The macroscopic observables form a commutative algebra $\mathcal{C}$ which is contained in the centre of $\mathcal{A}''$, i.e. $\mathcal{C} \subset \mathcal{Z} = \{Z \in \mathcal{A}''|[Z, A] = 0\,\forall A \in \mathcal{A}''\}$, yet is almost disjoint from the microscopic observables: $\mathcal{C} \cap \mathcal{A} = \mathbb{C}\mathbf{1}$. Transfer of information to a macroscopic observable therefore implies perfect decoherence on all microscopic and macroscopic observables (cf section 3.1.3).

Unfortunately, this transfer cannot be done by any automorphic time evolution, since the macroscopic observables are central. Hepp proposed information transfer by a $t \to \infty$ limit of automorphisms. He was able to show that this causes decoherence in the weak-operator sense. That is, on each fixed microscopic observable, the coherence becomes arbitrarily small for sufficiently large t.

The paper was criticized by John Bell a few years later [Bel], on the grounds that for each fixed time t, there are observables to be found on which coherence is not small. Since Bell was of the opinion that a 'wave packet reduction', even on closed systems, 'takes over from the Schrödinger equation', this was not to his satisfaction. He did agree however that these observables would become arbitrarily difficult to observe in practice for large t.

By considering large but finite closed systems subject to unitary time evolution, we hope to clarify the role that macroscopic systems play in making us mistake coherent superpositions for classical mixtures. It seems striking that the same, simple mathematics can also be used to understand why open systems do undergo decoherence as soon as they lose information.

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